2022

PHYSICS — HONOURS

Paper : CC-11

(Syllabus : 2019-2020)

[Electromagnetic Theory]

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***question no.*** 1 and ***any four*** questions from the rest.

1. Answer ***any five*** questions : 2×5

   (a) Find the velocity of light in a medium for which relative permittivity is 3 and permeability is half of the value of permittivity.

   (b) The intensity of sunlight hitting the earth is about 1300 Watt/m^2. Find the amplitude of the electric field.

   (c) Show that the frequency of the wave remains unchanged upon refraction, when an electromagnetic wave is incident on the plane interface between two different media.

   (d) Calculate the reflection coefficient for an electromagnetic wave of frequency 10 GHz when it is incident normally on a metal surface of conductivity 6×10^7 $(\Omega\text{m})^{-1}$. Take $\epsilon = \epsilon_0$ and $\mu = \mu_0$.

   (e) In a current free region $B_x = ax + bz$ and $B_y = ax + cy$. Find a possible form for B_z assuming all currents are outside.

   (f) Define specific rotation.

   (g) Define the state of polarization of the wave presented by the following set of equations $E_x = E_o \sin(kz - \omega t)$ and $E_y = E_o \cos(kz - \omega t)$.

2. (a) Show how Maxwell's equations in free space imply local conservation of charge. Show that this implies Kirchoff's second law.

   (b) Starting from Maxwell's equation derive the following equations :

   $$\left(\nabla^2 \vec{A} - \mu_o\epsilon_o \frac{\partial^2 \vec{A}}{\partial t^2}\right) - \vec{\nabla} L = -\mu_o \vec{J} \text{ and } \left(\nabla^2 \Phi - \mu_o\epsilon_o \frac{\partial^2 \Phi}{\partial t^2}\right) + \frac{\partial L}{\partial t} = -\frac{\rho}{\epsilon_o},$$ where symbols have their usual meanings. Comment on the term L. Hence derive modified Poisson equation.

   (2+2)+(3+2+1)

3. (a) A long coaxial cable of length l consists of an inner conductor of radius a and an outer conductor of radius b. It is maintained at a potential difference V. The current flows down the inner cylinder and back along the outer cylinder. If λ be the charge per unit length of the inner cylinder, find the electromagnetic energy stored in the cable.

(b) Show that the momentum density stored in electromagnetic field is given by $\vec{S}/c^2$ in vacuum; where $\vec{S}$ is the Poynting vector.

(c) Show that if a monochromatic linearly polarized plane wave is moving in an isotropic non-conducting medium, the time average of its energy density is distributed equally between the magnetic and electric fields. 3+4+3

4. (a) Determine the expression of the phase angle in a conductor lagged by the magnetic field behind the electric field. Determine the angle for a ideal conductor.

(b) Define a good and a bad conductor from the point of view of the frequency of the incident electromagnetic wave. A metal has conductivity of $7.2\times10^7\ (\Omega\,\mathrm{m})^{-1}$. Obtain the skin depth in that metal at frequency of 1 kHz. Why are metals opaque? (3+2)+(2+2+1)

5. (a) A current sheet of surface density $\vec{K} = -10\hat{y}\ A/m$ is located in the plane $z = 0$. The permiabilities of the meterial for $z > 0$ and $z < 0$ are $5\mu_o$ and $2\mu_o$ respectively. The magnetic field in the region $z > 0$ is $\vec{H}_> = 15\hat{x} - 8\hat{z}\ A/m$. Find $\vec{B}_>, \vec{H}_<$ and $\vec{B}_<$, where

$\vec{H}_>, \vec{B}_>$ stand for fields at $z > 0$, and

$\vec{H}_<, \vec{B}_<$ stand for fields at $z < 0$.

(b) A plane electromagnetic wave falls obliquely on the interface between two simple dielectrics. The electric field vector is parallel to the plane of incidence. Obtain expressions for the transmission coefficient.

(c) A plane EM wave travelling in free space is incident normally on a dielectric material with dielectric constant of 2.5. If there is no absorption by the material, find the reflectivity and transmittivity. 3+4+3

6. (a) Explain how to detect the following waves :

(i) circularly polarized, (ii) elliptically polarized.

(b) What is Babinet's compensator? Write the advantages of Babinet's compensator over the $\lambda/4$ plate.

(c) Explain how continuous variation in retardance of a wave plate can be achieved.

(d) Calculate the thickness of a half-wave plate for a light of wavelength 550 nm. Given the refractive indices $n_o = 1.5442$ and $n_e = 1.5533$. 2+(2+1)+2+3

7. (a) Imagine two crossed linear polarizers with transmission axes vertical and horizontal. Now insert a third linear polarizer between them with transmission axis at 45° to the vertical. Determine the intensity of the emerging wave in terms of intensity of the incident wave.

(b) The rotation in the plane of polarization at wavelength 589.3 nm in a certain substance is 10°/cm. Calculate the difference between the refractive indices for right and left circular polarized light in the medium. Derive the equation that you use.

(c) Define Optic axis, Principal section and Principal plane for a double refracting crystal. 3+4+3

Syllabus : 2018-2019

(Quantum Mechanics and Applications)

Full Marks : 50

Answer ***question no.*** 1 and ***any four*** questions from the rest.

1. Answer ***any five*** questions : 2×5

(a) A particle moves in a linear potential $V(x) = (-kx)$. Show that $\langle x \rangle$ follows the same time evolution as given by Newton's law.

(b) Prove that total wave function for Bosons is symmetric.

(c) In Stern-Gerlach experiment, a beam of atoms is passed through an inhomogeneous magnetic field. What will happen if ions are used instead of atoms.

(d) If $V(x)$ is an even function, then wave function can always be taken to be either even or odd. Justify.

(e) Show that the quantity $y = \sqrt{\alpha}x$, where $\alpha = \frac{m\omega}{\hbar}$ as used in a harmonic oscillator is dimensionless.

(f) Find the magnetic moment of an atom in the state $2D_{3/2}$.

(g) Find the quantum number corresponding to an oscillator of mass 2 gm, angular velocity 1 rad/Sec, amplitude 1 cm & using correspondence principle, comment on its nature.

2. (a) A stream of particles of mass m energy E move towards the potential step $V(x) = 0$ for $x < 0$ and $V(x) = V_0$ for $x > 0$. If the energy of the particles $E > E_0$, show that the sum of fluxes of the transmitted and reflected particles is equal to flux of incident particles.

(b) The wave function of linear harmonic oscillator in the potential $V(x) = \frac{1}{2}mw^2x^2$ at a given instant is $\psi(x) = \left(\frac{2}{\pi\beta^2}\right)^{\frac{1}{4}} e^{-x^2/\beta^2}$ For what value of β will the function $\hat{a}\,\psi(x)$ be zero?

[Given : $\hat{a} = \frac{1}{\sqrt{2}}\left[\hat{x} + i\hat{p}_x\right]$

(c) Prove that energy eigenfunction of a free particle is doubly degenerate. 5+3+2

3. (a) The vector space for a particle of spin half has basis $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$ in which s_z is diagonal having eigenvalues ½ and –½ respectively. Using this basis find the normalized eigenvector of S_y with eigenvalues (–½).

(b) Consider the wave function

$$\psi(x,0) = Ae^{-\lambda|x|}e^{-iwt}$$

(i) Normalize ψ

(ii) Determine the expectation value of x.

(iii) Find the standard deviation of x.

(c) Discuss the changes that the confined particle observes if the infinite square will is made finite. 3+(1+2+2)+2

4. (a) Write down the schrödinger equation for the electron of Tritium (H_3) atom, assuming the nucleus to be stationary. Obtain the radial equation by separation of variables with special emphasis on effective potential.

(b) Find $\langle S_z \rangle$ in the state $X = A\begin{pmatrix}1-2i\\2\end{pmatrix}$.

(c) Show that $L_{\pm}f_l^m = \hbar\sqrt{l(l+1) - m(m+1)}\,f_l^{m\pm1}$ where the symbols have their usual meaning. 4+3+3

5. (a) What is Lande-g factor? Obtain an expression in terms of l, s, j.

(b) What is the lande-g-factor for an atom with a single optical electron in $d_{3/2}$ level?

(c) Find out the different factors in the J-J coupling. Scheme for a two electron atom given by $l_1 = 1$ and $l_2 = 2$. (1+3)+2+4

6. (a) Write down the fine structure formula mentioning each correction term with respect to Bohr energy of hydrogen atom.

(b) Using 1st order perturbation theory, obtain the relativistic correction term to the kinetic energy.

(c) Using uncertainty principle estimate the ground state energy of hydrogen atom. 3+4+3

7. (a) An atomic state is denoted by $4D_{5/2}$. Give the values of L, S and J. What should be the minimum number of electrons involved for this state? Give a possible electronic configuration.

(b) In K, the longest wavelength lines in $(n, 1) \rightarrow (4, 0)$ transitions have wavelength 7699, 7665, 4047 and 4044 Å respectively. Find the splitting between the levels in the same n and l but different j.

(c) A source of light is placed between the poles of an electromagnet. What will you observe if the light is examined by a spectroscope in directions parallel and perpendicular to the magnetic field. Assume the magnetic field to be not too strong. Give an explanation of the phenomenon. 4+3+3