2021

PHYSICS — HONOURS

[2018-19 and 2019-20 Syllabus]

Paper : CC-3

(Electricity and Magnetism)

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***question no.*** **1** and ***any four*** questions from the rest.

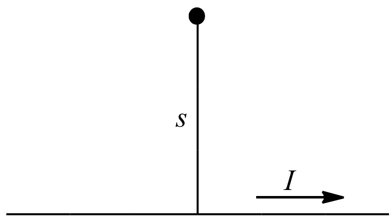
1. Answer ***any five*** questions : 2×5

(a) Consider a sphere of radius R around the origin surroundings n charges where jth charge, e_j is located at points $\vec{r_j}$, with $r_j < R$. Express total charge density of the sphere in terms of delta function and evaluate the total electric flux $\oint_S \vec{E}\cdot d\vec{s}$ coming out of the sphere.

(b) Find the general solution to Laplace's equation in spherical coordinates, for the case where the potential, V depends on r only.

(c) 'A dielectric medium is said to be simple if it is homogeneous, isotropic and linear.'– Explain the statement in brief. Find a relation between susceptibility and permittivity for such a medium.

(d) Find the magnetic field at a distance s due to an infinitely long, straight filament of current I.



(e) Show that Faraday's law of induction is equivalent to the equation $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$.

(f) Derive an expression for the speed of light in vacuum from Maxwell's equation.

(g) 'Lenz's law is a consequence of the law of conservation of energy' — Explain.

2. (a) Write the expression of potential energy of a dipole with dipole moment $\vec{p}$ placed in an external electric field $\vec{E}$. Hence show that the force on the dipole due to the electric field is given by $\vec{F} = \left(\vec{p}.\vec{\nabla}\right)\vec{E}$.

(b) Define electric displacement vector in a dielectric medium. Show that the curl of the displacement vector is not zero in general even in electrostatic case.

(c) Show that the electric potential at any arbitrary point $\vec{r}$ due to a electric dipole placed at $\vec{r}\,'$ with dipole moment $\vec{p}$ can be written as $\phi(r) = -\vec{p}.\vec{\nabla}\phi_0$, where ϕ_0 is the potential at $\vec{r}$ due to a unit positive point charge placed at $\vec{r}'$. Assume that the length of the dipole is very small compared to $\left|\vec{r} - \vec{r}\,'\right|$. (1+2)+(1+1)+5

3. (a) Consider a point charge at the origin. Starting from the equation :

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

show that $\vec{\nabla} \times \vec{E} = \vec{0}$.

(b) The electric field in a region is given by $\vec{E} = kr^3\hat{r}$. Calculate the amount of charge contained within a spherical surface of radius a centred at the origin.

(c) Suppose a charge Q is distributed within a sphere of radius R in such a way that the charge density $\rho(r)$ at a distance r from the centre of the sphere is

$$\rho(r) = \begin{cases} K(R-r) & \text{for } 0 < r < R \\ 0 & \text{for } r \geqslant R \end{cases}$$

(i) Determine constant K in terms of Q and R.

(ii) Calculate the electric field at any point inside the sphere.

(iii) Find the value of r for which the field is maximum.

(iv) What is the value of this maximum field? 2+3+(1+2+1+1)

4. (a) A sphere of radius R carries polarization

$$\vec{P}\left(\vec{r}\right) = K\vec{r}.$$

where K is a constant and $\vec{r}$ is the radius vector from the centre.

(i) Calculate the bound charge densities σ_b and ρ_b.

(ii) Find the field inside and outside of the sphere.

(b) A point charge q is placed at a distance d from the centre of a grounded conducting sphere of radius "a" ($a < d$).

(i) Calculate the radial component of the electric field at a distance r from the centre ($r > a$).

(ii) Find the induced surface charge density on the sphere. (2+2)+(3+3)

5. (a) Starting from Biot-Savart law derive Ampere's Circuital law.

(b) A steady current I flows down a long cylindrical wire of radius a. Find the magnetic field, both inside and outside the wire, if

(i) the current is uniformly distributed over the outside surface of the wire

(ii) the current is distributed in such a way that the current density J is proportional to s, the distance from the axis.

(c) A long straight conductor carries a current I. Determine the force per unit length on the conductor when it is placed in a uniform magnetic field. 4+(2+2)+2

6. (a) Show that the equivalent inductance of two coils of self inductances L_1, L_2 and Mutual inductance M connected in parallel is :

$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 \pm 2M}.$$

(b) A coil of self inductance 100 mH is connected in series with another coil of self inductance 169 mH. The effective inductance of the combination is found to be 70 mH. Determine the coefficient of coupling.

(c) Consider a parallel L-C combination in series with a resistance R. Calculate the output voltage across L-C combination for sinusoidal input. 4+3+3

7. (a) What do you mean by displacement current? Show that Maxwell's equations are consistent with the equation of continuity.

(b) Consider plane wave solutions of electromagnetic wave equation in free space. Show that the wave is transverse in nature.

(c) A 100W, 200V electric lamp has to be connected to the AC mains with rms voltage 400V and frequency 50Hz. What capacitor must be added in series with the lamp to obtain the normal glow for which the lamp has been constructed? (1+2)+(2+1)+4

Or,

(Only for 2018–2019 Syllabus)

(c) Consider the circuit below. Using the principle of superposition find the potential drop across the 5kΩ resistor. 4