2018

PHYSICS – HONOURS

Fifth Paper

Full Marks – 100

The figures in the margin indicate full marks

Candidates are required to give their answers in their own words as far as practicable

1. Answer ***any ten*** of the following questions : 2×10

(a) Define holonomic and non-holonomic constraints with an example of each.

(b) If the Lagrangian is given by

$$L(x,\dot{x}) = \frac{\dot{x}^2}{2x} - V(x)$$

What will be the corresponding Hamiltonian?

(c) An U-tube contains a zero-viscosity fluid up to a length L. If one end of the fluid column is depressed by a little distance, show that the fluid column oscillates with time period, $T = 2\pi\sqrt{\frac{L}{2g}}$; g being the acceleration due to gravity.

(d) The momentum of a body becomes four times when its speed doubles. What was the initial speed of the body in units of c?

(e) Prove that the four dimensional volume element $dx\ dy\ dz\ dt$ is invariant under Lorentz transformation.

(f) If A_i and B_j are arbitrary covariant vectors and $C^{ij}A_iB_j$ is a scalar, prove that C^{ij} is a contravariant tensor of second rank.

(g) Commutator of two matrices A and B is defined by [A, B] = AB – BA and the anti- commutator by {A, B} = AB + BA. If {A, B} = 0, find [A, BC].

(h) Can we measure the kinetic and potential energies of a particle simultaneously with arbitrary precision?

(i) Show that the commutator $[x, [x, H]] = -\frac{\hbar^2}{m}$, where H is the Hamiltonian operator.

(j) Calculate the angle between the total $(\bar{J})$ and orbital angular $(\bar{L})$ momentum vectors for an electron in ${}^4D_{3/2}$ state.

(k) Which of the following substances can give rise to pure rotation-vibration spectra?

H_2, HF, O_2, CO

(l) What is the importance of presence of a metastable state in lasing action?

Group – A

Section – I

Answer ***any two*** questions

2. (a) Show that the effective potential of a particle of mass m in a central force field is given by

$$U_{eff}(r)=U(r)+\frac{L^2}{2mr^2}$$

where L is the angular momentum. 3

(b) A particle of mass m moving in a central force field describes a spiral orbit $r=k\theta^2$ where k is a positive constant.

(i) Find the force law. Given that the differential equation of the orbit is

$$\frac{L^2}{mr^2}\left(\frac{d^2\left(\frac{1}{r}\right)}{d\theta^2}+\frac{1}{r}\right)=-f(r).$$

(ii) Compute the effective one-dimensional potential energy.

(iii) Find the total energy of the system, for which this motion is allowed. 2+3+2

3. (a) What do you mean by generalized forces? Find its expression in terms of generalized coordinates. 1+2

(b) A particle of mass m is constrained to move on the surface of a smooth sphere of radius R. There are no external forces of any kind acting on the particle.

(i) Choose a set of generalized coordinates and write the Lagrangian of the system.

(ii) Derive the Hamiltonian of the system. Is it conserved?

(iii) Using the Hamiltonian equations of motion, prove that the motion of the particle is along a great circle of the sphere. [A great circle on a sphere is a circle on the sphere's surface whose center is the same as the center of the sphere]. 2+(2+1)+2

4. (a) Consider the Lagrangian

$$L=\frac{1}{2}m\left(\dot{x}^2-\omega^2x^2\right)e^{\gamma t}$$

for the motion of a particle of mass m in one-dimension (x). The constants m, γ and ω are real and positive. Construct the Hamiltonian. Is the Hamiltonian a constant of motion? 3+1

(b) Show that $q_1 q_2$ is a constant of motion for the Hamiltonian $H = q_1 p_1 - q_2 p_2 - a q_1^2 + b q_2^2$. 2

(c) Establish Bernoulli's equation of fluid motion stating the assumptions used. 4

Section – II

Answer ***any two*** questions

5. (a) From the basic definition of space-time interval, explain with suitable diagrams (i) a space-like interval, and (ii) a time-like interval for a two-dimensional space-time geometry. 2+2

(b) In a certain inertial frame light pulses are emitted by two sources 5 km apart. Time interval between two pulses is $5\,\mu s$. An observer moving at a speed V along the line joining these sources notes that the pulses are simultaneous. Find the speed V of the observer. 3

(c) Two rockets of rest length L_0 are approaching each other from opposite directions at same speed $\frac{c}{2}$. How long does one of them appear to the other? 3

6. (a) Define proper time interval and show that it is a Lorentz invariant quantity. Hence construct velocity four-vector and show that it is a time like vector. 1+1+1+1

(b) A neutral pion of rest mass m and relativistic momentum $P = \frac{3}{4} mc$ decays into two photons. One of the photons is emitted in the same direction as the original pion, and the other in the opposite direction. Find the relativistic energy of each photon. 4

(c) Discuss about inconsistency, if any, in Newton's law of gravitation in the light of postulates of special theory of relativity. 2

7. (a) Defining $A_j = g_{jk} A^k$ and $A^k = g^{jk} A_j$ where symbols bear usual meaning, write down the mathematical relationship between g^{jk} and g_{jk}. What are the signed values of g^{ij} and g_{ij} in case of a 3 dimensional flat space time (2 space and 1 time dimension)? 2+2

(b) What is the space-time interval between any two events on the locus of a photon in space-time geometry; consistent with the definition $ds^2 = g_{ij} dx^i dx^j$? How does this interval vary from one inertial frame to another? Explain your answer with justification. 2

(c) A particle of rest mass m_0 moving with speed V collides and sticks with a stationary particle of rest mass M_0. Show that the speed of the composite particle is given by

$$\gamma m_0 V / (M_0 + \gamma m_0), \text{ where } \gamma = \left(1 - \frac{V^2}{C^2}\right)^{-\frac{1}{2}}.$$ 4

Group – B

Section – I

Answer ***any two*** questions

8. (a) Consider the potential $V(x) = \begin{cases} 0, & 0 < x < a \\ \infty, & \text{elsewhere} \end{cases}$.

(i) Estimate the energies of the ground state and the first excited state for an electron enclosed in a box of size $a = 10^{-10}$ m.

(ii) Calculate the same energies for a 1g metallic sphere which is moving in a box of size a = 10 cm.

(iii) From results of (i) and (ii) discuss why quantum mechanical effects are not important in second case. 4+2+1

(b) For the above potential, the wave function of a particle in the position space is given by $\phi(x) = \sqrt{\frac{2}{a}} \sin\frac{2\pi x}{a}$. Determine its momentum space wave function. 3

9. (a) Show that for stationary states in quantum mechanics variance of H (Hamiltonian) is zero. 2

(b) A potential barrier of height V_0 extends from $x = -a$ to $x = +a$. Prove that for a particle of energy $E < V_0$, the transmission coefficient through the barrier is given by

$$T^{-1} = 1 + \frac{V_0^2}{4E(V_0 - E)} \sinh^2\left(\frac{2a}{\hbar}\sqrt{2m(V_0 - E)}\right)$$ 5

(c) The ground state wave function of a particle of unit mass moving in a one-dimensional potential $V(x)$ is $e^{-\frac{x^2}{2}} \cosh\left(\sqrt{2}x\right)$. Find the potential $V(x)$, in suitable units in which $\hbar = 1$. 3

10. (a) A harmonic oscillator has a normalized wave function $\psi(x) = \frac{1}{\sqrt{3}}\psi_2(x) + c\psi_7(x)$ where $\psi_n(x)$ are the energy eigen-states. What is the magnitude of the constant c? Hence find the expectation value of the energy in this state. 2+2

(b) Write down Pauli's spin matrices σ_x, σ_y and σ_z. The eigen-vectors of the operator σ_z are $|\alpha\rangle$ and $|\beta\rangle$. Show that $\frac{|\alpha\rangle+|\beta\rangle}{\sqrt{2}}$ and $\frac{|\alpha\rangle-|\beta\rangle}{\sqrt{2}}$ are the normalised eigen-vectors of σ_x. 2+2

(c) Evaluate the following commutator :

$$\left[\bar{L}.\bar{S}, J^2\right].$$ 2

Section – II

Answer ***any two*** questions

11. (a) Calculate the Lande' g factors for $^3S_1$ and $^3P_1$ levels. Hence estimate the energy splitting of the two levels if a magnetic field of 1 T is applied. How many spectral lines will arise from the anomalous Zeeman splitting due to transition between these levels? Draw a neat diagram showing these transitions. (One Bohr Magneton = 9.27×10^{-24} J/T) 2+2+3

(b) The $J=0\rightarrow J=1$ rotational absorption line occurs at 1.153×10^{11} Hz in $C^{12}O^{16}$ and at 1.102×10^{11} Hz in $C^{x}O^{16}$. Find the mass number of the unknown carbon isotope (C^x). 3

12. (a) Into how many fine structure lines each line of the Balmer series of Hydrogen will split due to spin-orbit coupling? Justify your answer. 4

(b) Consider a two-electron system with $l_1=2$ and $l_2=1$. What are the possible total angular momentum J states, assuming LS coupling? Write the spectral term for each state. 4

(c) In a Stern-Gerlach experiment, always a beam of neutral atoms is used, and not ions. Explain the reason. 2

13. (a) Show that in a 3-level system it is possible to produce the required population inversion using a beam of suitable intensity. What is inversion threshold pumping rate? 4+2

(b) In a He-Ne laser, transition from 3S to 2P level gives a laser emission of wavelength 632.8 nm. If the 2P level has energy equal to 15.2×10^{-19} J, assuming no loss, calculate the pumping energy required. 2

(c) Why do molecules show band spectra rather than line spectra? 2