2017

PHYSICS — HONOURS

Second Paper

(Group – A)

Full Marks – 50

The figures in the margin indicate full marks

Candidates are required to give their answers in their own words as far as practicable

Answer ***Question No. 1*** and ***any four*** from the rest

1. Answer ***any five*** of the following : 2×5

(a) A particle is under the influence of a force $\vec{F}$ and has an instantaneous velocity $\vec{V}$. Show that $\frac{dT}{dt} = \vec{F}.\vec{V}$, where T is the kinetic energy.

(b) The angular momentum of a particle is $\vec{l}(t) = f(t)\hat{n}$ where f(t) is a function of time and $\hat{n}$ is some constant unit vector. Show that the motion of this particle is restricted to a plane.

(c) "Angular momentum and angular velocity of a rigid body are not always parallel" — Justify the statement.

(d) What is the significance of the total area under the Maxwellian velocity distribution curve?

(e) Write down Planck's law for the energy distribution of black body radiation. Plot the distributions at two different temperatures.

(f) How much time will it take for a layer of ice of thickness 20 cm to increase by 10 cm on the surface of a pond when the temperature of the surrounding is – 15°C? Given K = 0.005 C.G.S. unit, L = 80 cal/gm, ρ = 0.9 gm/cm^3.

2. A particle of mass m moves along a trajectory given by $x = x_0 \cos\omega t$, $y = y_0 \sin\omega t$.

(a) Find the x and y component of the force. What is the condition under which the force is a central force? 2+2

(b) Find the potential energy as a function of x and y. 2

(c) Determine the kinetic energy of the particle. Show that the total energy of the particle is conserved. 2+2

3. (a) For a system of particles of total mass M acted upon by total external force $\vec{F}_{ext}$, show that $\vec{F}_{ext} = M\vec{a}$, where $\vec{a}$ is the acceleration of the centre of mass. 3

(b) A uniform cylindrical firecracker of mass 'm' is ignited and projected from a height 'h' (centre of mass height) with an initial horizontal velocity $\vec{u}_0$. The firecracker explodes after time 't'_e' in mid-air. Determine the path of the centre of mass. 2

(c) If the centre of mass of a system is accelerating, show that the total torque about the centre of mass due to the pseudoforce arising out of the acceleration of the centre of mass is zero.

Hence, show that for a system of particles

$$\vec{T} = \frac{d\vec{L}'}{dt}$$

[*Turn Over*]

with $\vec{T} = \sum_i \vec{r}_i' \times \vec{F}_i$ and $\vec{L}' = \sum_i \vec{r}_i' \times \vec{p}_i'$. Here, $\vec{r}_i'$ and $\vec{p}_i'$ are the position and momentum of the i-th particle with respect to the centre of mass but $\vec{F}_i$ is the true total force on the i-th particle as seen from an inertial frame. Under what condition can you replace the torque in the above equation by the external torque on the system? 2+2+1

4. (a) Find the moment of inertia of a rigid body about an arbitrary axis having direction cosines α_1, α_2 and α_3 with respect to X, Y and Z axes, respectively and passing through the origin. What is ellipsoid of inertia? 4+2

(b) A homogeneous square lamina is placed on the x-y plane with its centre at the origin and edges parallel to the x and y axes. Now the coordinate system is rotated by some angle ϕ about the z-axis. Show that the resulting coordinate system is a principal axes system irrespective of the angle ϕ. 4

5. (a) The relative motion of two molecules can be described as the motion of a single particle of reduced mass $m_r = \frac{m}{2}$. Using this idea we get the distribution of the relative velocities of the molecules

$$^{dn}C_r = n\left(\frac{m}{4\pi KT}\right)^{3/2} \exp\left(-\frac{mC_r^2}{4KT}\right) 4\pi C_r^2 dC_r$$

Where C_r is the relative velocity of two molecules. Show that the average relative velocity of the molecules is $\overline{C}_r = \sqrt{2}\,\overline{c}$, where $\overline{c}$ is the average velocity of the molecules. 4

(b) A shower of 5000 molecules of a gas, each moving with the same velocity initially, traverses a gas. Find the number of molecules that would travel undeflected even after travelling a distance equal to the mean free path. Also, find the number of molecules having free path lying within λ and 2λ, where λ is the mean free path. 2+1

(c) Prove that coefficient of viscosity of a gas is independent of pressure over a wide range. Why is this no longer valid at extremely low pressure? 2+1

6. (a) Colloidal particles are suspended in a liquid. Using Einstein's theory, find the temperature dependence of the mean square displacement per unit time. 6

(b) A particle under Brownian motion at 27°C has an r.m.s speed 1 m/sec. Find the mass of the particle. Boltzmann Constant $K = 1.38 \times 10^{-23}$ J/K. 4

7. (a) Express van der Walls equation of state of a real gas in virial form. Determine the Boyle temperature from it. 2+1

(b) A long wire of resistivity 2×10^4 ohm – cm and 1 mm in diameter carries a current of 5 ampere. If it is covered uniformly with a cylindrical layer of insulating material having coefficient of thermal conductivity of 6×10^{-4} c.g.s unit and outer diameter of 1 cm, what is the temperature difference between the inner and outer surface of the insulating layer at steady state? 4

(c) What is solar constant? Each square meter of sun's surface radiates energy at the rate of 6.3×10^7 Joule/m²/s, and Stefan's Constant is 5.67×10^{-8} W/m²/K⁴. Find the temperature of the surface of the sun. 1+2